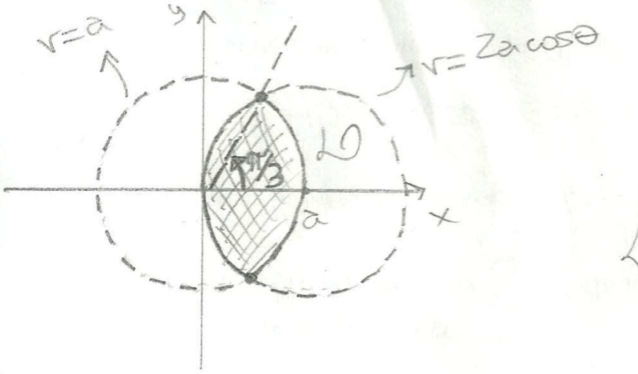
3. (2.0) Calcule a área comum aos discos $0 \leq r \leq a$ e $0 \leq r \leq 2a cos\theta$.

[Dica: $cos^2\theta = \frac{1}{2} + \frac{1}{2}cos2\theta$]

$$r = 2a\cos\theta \Leftrightarrow r^2 = 2ar\cos\theta \Leftrightarrow x^2+y^2 = 2ax \Leftrightarrow x^2-2ax+y^2=0 \Leftrightarrow$$

$$\Leftrightarrow x^2-2ax+a^2+y^2 = a^2 \Leftrightarrow \boxed{(x-a)^2+y^2=a^2}$$

Circunferência centrada no ponto $(a,0)$ e raio a.



Intersecção das 2 circunferências:

$$\begin{cases} r=a \\ r=2a\cos\theta \end{cases} \Rightarrow \cancel{a} = 2\cancel{a}\cos\theta \Rightarrow$$

$$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow$$

$$\Rightarrow \theta = \pm\frac{\pi}{3}$$

Por simetria, temos:

$$\text{Área}(D) = 2\left(\frac{1}{2}\int_0^{\pi/3} a^2\, d\theta + \frac{1}{2}\int_{\pi/3}^{\pi/2} (2a\cos\theta)^2\, d\theta\right) =$$

$$= \int_0^{\pi/3} a^2\, d\theta + \int_{\pi/3}^{\pi/2} 4a^2\cos^2\theta\, d\theta = a^2\int_0^{\pi/3} 1\, d\theta + 4a^2\int_{\pi/3}^{\pi/2} \cos^2\theta\, d\theta$$

$$= \frac{\pi a^2}{3} + 4a^2\int_{\pi/3}^{\pi/2} \frac{1}{2}+\frac{1}{2}\cos2\theta\, d\theta = \frac{\pi a^2}{3} + 4a^2\int_{\pi/3}^{\pi/2}\frac{1}{2}\, d\theta +$$

$$+ 2a^2\cdot\int_{\pi/3}^{\pi/2} \cos2\theta\, d\theta = \frac{\pi a^2}{3} + 2a^2\left(\frac{\pi}{2}-\frac{\pi}{3}\right) + 2a^2\left(\frac{\operatorname{sen}2\theta}{2}\Big|_{\pi/3}^{\pi/2}\right) =$$

$$= \frac{\pi a^2}{3} + \frac{\pi a^2}{3} + 2a^2\left(\cancelto{0}{\frac{\operatorname{sen}\pi}{2}} - \frac{\operatorname{sen}(2\pi/3)}{2}\right) =$$

$$= \frac{2a^2\pi}{3} + 2a^2\left(-\frac{\sqrt{3}}{4}\right) = \boxed{\frac{2a^2\pi}{3} - \frac{\sqrt{3}}{2}a^2}$$