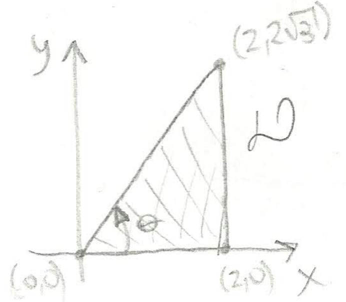
2. (2.0) Usando coordenadas polares, calcule $\int_{\mathcal{D}}\int xy\, dA$, onde $\mathcal{D}$ é a região triangular de vértices $(0,0)$, $(2,0)$ e $(2, 2\sqrt{3})$.



$$tg\theta = \frac{2\sqrt{3}}{2} \Rightarrow tg\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$$

Para o segmento $x=2$, temos:

$$x = 2$$
$$r\cos\theta = 2$$
$$r = \frac{2}{\cos\theta}$$

Portanto, $\mathcal{D}: \begin{cases} 0 \leq \theta \leq \frac{\pi}{3} \\ 0 \leq r \leq \frac{2}{\cos\theta} \end{cases}$

$$\iint_{\mathcal{D}} xy\, dA = \int_0^{\pi/3}\int_0^{\frac{2}{\cos\theta}} \underbrace{(r\cos\theta)}_{x}\underbrace{(r\,sen\theta)}_{y}\underbrace{r}_{jacobiano}\, dr\, d\theta = \int_0^{\pi/3}\int_0^{\frac{2}{\cos\theta}} r^3 \cos\theta\, sen\theta\, dr\, d\theta$$

$$= \int_0^{\pi/3} \frac{r^4}{4}\, sen\theta \cos\theta\, dr\, d\theta = \int_0^{\pi/3} \frac{\left(\frac{2}{\cos\theta}\right)^4}{4}\, sen\theta\cos\theta\, d\theta =$$

$$= \int_0^{\pi/3} \frac{\frac{16\, sen\theta}{\cos^3\theta}}{4}\, d\theta = \int_0^{\pi/3} \frac{4}{\cos^3\theta}\, sen\theta\, d\theta \overset{(*)}{=}$$

$$\overset{(*)}{=} \frac{2}{\cos^2\theta}\Bigg|_0^{\pi/3} = \frac{2}{\cos^2\frac{\pi}{3}} - \frac{2}{\cos^2 0} = \frac{2}{\left(\frac{1}{2}\right)^2} - \frac{2}{1} = \frac{2}{\frac{1}{4}} - 2 = 8 - 2 = \boxed{6}$$

(*) Fazendo uma mudança de variável para achar a primitiva:

$u = \cos\theta \Rightarrow du = -sen\theta\, d\theta \Rightarrow sen\theta\, d\theta = -du$

Então

$$\int \frac{4}{\cos^3\theta}\, sen\theta\, d\theta = \int \frac{4}{u^3}(-du) = -4\int u^{-3}\, du = -4\,\frac{u^{-3+1}}{-3+1} = 2u^{-2} = \frac{2}{u^2}$$

$$= \frac{2}{\cos^2\theta} + C$$