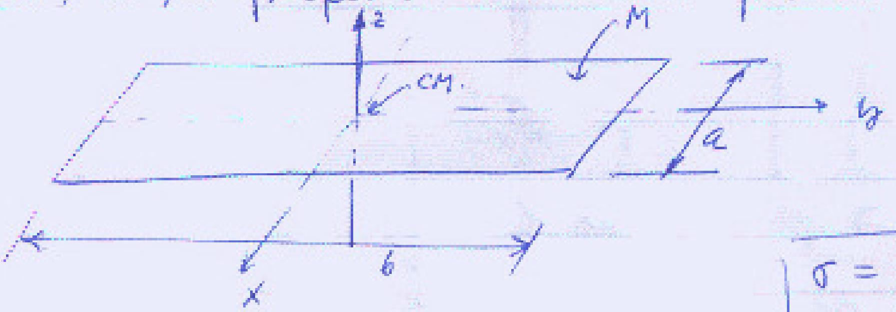
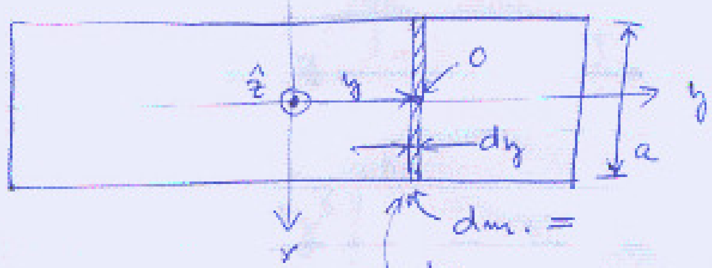
Momento de Inércia de Placa Retangular

(a) eixo perpendicular à placa



$$\boxed{\sigma = \frac{M}{ab}} \leftarrow \text{área}$$



$dm =$

$$dI_{barrinha,0} = \frac{dm\, a^2}{12} = dI_0$$

$$dI_z = dI_{barrinha,z} = \underline{\overline{dI_0 + dm\, y^2}} \quad \text{T. eixo paralelo}$$

$$dI_z = \frac{dm\, a^2}{12} + dm\, y^2 = dm\left(\frac{a^2}{12} + y^2\right)$$

$$dm = a\, dy \cdot \sigma \Rightarrow dI_z = \sigma a\, dy \left(\frac{a^2}{12} + y^2\right)$$

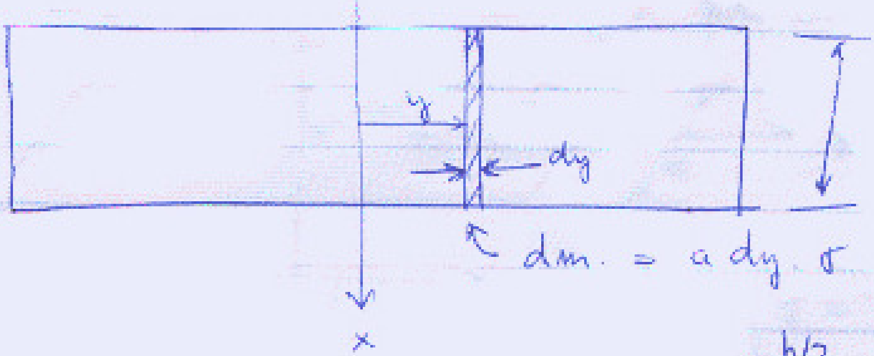
$$I_z = \int_{y=-b/2}^{b/2} \sigma a\, dy \left(\frac{a^2}{12} + y^2\right) = \sigma a \cdot \left[\frac{a^2}{12}\int_{-b/2}^{b/2} dy + \int_{-b/2}^{b/2} y^2 dy\right]$$

$$I_z = \sigma \cdot a\left[\frac{a^2}{12} \cdot b + \frac{y^3}{3}\Big|_{-b/2}^{b/2}\right] =$$

$$I_z = \sigma \cdot a\left[\frac{a^2 b}{12} + \frac{b^3}{12}\right] = \frac{M}{\cancel{a}b}\cancel{a}\left[\frac{a^2 b}{12} + \frac{b^3}{12}\right] =$$

$$\boxed{I_z = \frac{M}{12}\left[a^2 + b^2\right]}$$

(b) eixo no plano da placa. (eixo x)



$$dI_{\text{barrinha},x} = y^2 . dm \qquad I_x = \int_{y=-b/2}^{b/2} y^2 . a\sigma\, dy$$

$$I_x = a\sigma \left.\frac{y^3}{3}\right|_{-b/2}^{b/2} = \cancel{a} . \frac{M}{\cancel{a}b} . \frac{1}{3}\left[\frac{b^3}{8} + \frac{b^3}{8}\right]$$

$$\boxed{I_x = \frac{M}{3}\left[\frac{b^2}{4}\right] = \frac{Mb^2}{12}}$$

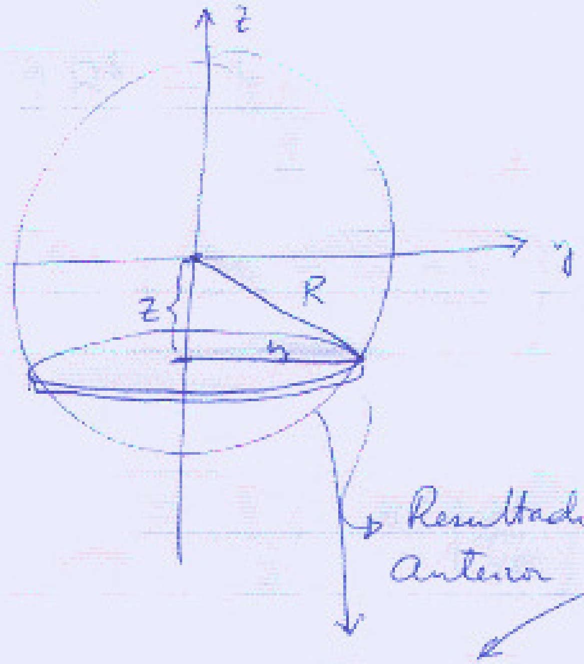
Obs. [mesmo que o de uma barra de comprimento b e massa M.

(b') eixo (y)

→ por analogia c/o acima.

$$\boxed{I_y = \frac{Ma^2}{12}}$$

Momento de Inércia de uma Esfera Maciça em torno do eixo diamentral.



$$R^2 = y^2 + z^2$$

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3}$$

Resultado anterior

$$I_{circulo} = \frac{m r^2}{2}$$

$$dI_{\substack{bolacha \\ circular,\, z}} = \frac{dm\, y^2}{2}$$

$$dm = \rho \underbrace{dV}_{A(y)\,dz} = \rho \cdot \pi y^2 . dz = \pi \rho \left(R^2 - z^2\right) dz$$

$$dI_{bolacha,\, z} = \frac{\pi \rho \left(R^2 - y^2\right) dz}{2} \cdot \left(R^2 - z^2\right)$$

$$I_{Esfera} = \int_{z=-R}^{R} dI_{bolacha,\, z} = \frac{\pi \rho}{2} \int_{z=-R}^{R} \left(R^2 - z^2\right)^2 dz$$

$$\int_{z=-R}^{R} \left(R^2 - z^2\right)^2 dz = R^4 (R + R) - 2R^2 \int_{-R}^{R} z^2 dz + \frac{z^5}{5}\Big|_{-R}^{R} =$$

$$R^4 - 2R^2 z^2 + z^4$$

$$= R^4 . 2R - 2R^2 . \frac{z^3}{3}\Big|_{-R}^{R} + \frac{R^5}{5} + \frac{R^5}{5} =$$

$$= 2R^5 - 2R^2\left[\frac{R^3}{3} + \frac{R^3}{3}\right] + \frac{2R^5}{5}$$

$$= 2R^5 - \frac{4R^5}{3} + \frac{2R^5}{5} = R^5\left[2 - \frac{4}{3} + \frac{2}{5}\right] =$$

$$= R^5\left[\frac{30 - 20 + 6}{5}\right] = R^5\left(\frac{16}{15}\right)$$

$$I_{Esfera} = \frac{\pi\rho}{\cancel{2}} \cdot R^5\left(\frac{\cancel{16}^{8}}{15}\right) = \cancel{\pi} \cdot \frac{M}{\frac{\cancel{4}}{\cancel{3}}\cancel{\pi}\cancel{R^3}} R^{\cancel{5}\,2} \cdot \frac{\cancel{8}^{2}}{\cancel{15}^{5}}$$

$$\boxed{I_{Esfera} = \frac{2}{5} M R^2}$$